

Transport Anomalies Associated with the Pseudogap From a Preformed Pair Perspective

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Transport studies seem to be one of the strongest lines of support for a preformed pair approach to the pseudogap. In this paper we provide a fresh, physically transparent look at two important quantities: the diamagnetic susceptibility and conductivity. We use a three dimensional preformed pair framework which has had some success in the cold Fermi gases and in the process we reconcile recently observed inconsistencies. Specifically, while the preformed pairs in our theory give a large contribution to the diamagnetic susceptibility, the imaginary part of the conductivity is suppressed to zero much closer to T_c , as is observed experimentally.

One of the biggest challenges in understanding the high temperature superconductors revolves around the origin of the ubiquitous pseudogap. Because this normal state gap has d -wave like features compatible with the superconducting order parameter, this suggests that the pseudogap is related to some form of “precursor pairing” which would generalize the behavior in conventional BCS superconductors, (where pairing and condensation take place at precisely the same temperature). On the other-hand, there are many reports [1, 2] suggesting that the pseudogap onset temperature is associated with a broken symmetry and, thus, another order parameter. It is widely believed that because the pseudogap has clear signatures in generalized transport, these measurements may help with the centrally important question of distinguishing the two scenarios. In this paper we provide a fresh, transparent look at transport in the presence of a pseudogap where the latter is associated with pre-formed pairs deriving from a stronger than BCS attractive interaction. We are thereby able to reconcile inconsistencies with cuprate experiments. Importantly, there is no more theoretical flexibility here than in standard BCS theory so that predictions are concrete and testable.

Our goal is to address the observed conflict between transport experiments [3, 4] and a variety of precursor superconductivity scenarios before reaching the definitive conclusion that the pseudogap derives from a non-superconducting order parameter. We argue here that it is necessary to investigate one more precursor superconductivity approach. Most importantly, this particular scenario, based on a stronger than BCS attraction, has been realized experimentally— in atomic Fermi gases [5] which also appear to exhibit a pseudogap [6–8]. We argue it should also be applicable to those superconductors (such as the cuprates) with anomalously high pairing onset temperature T^* , and small pair size. Similar ideas were introduced by Geshkenbein, Ioffe and Larkin [9]. In contrast to previous work here we discuss trans-

port both above and below the transition T_c and we pay central attention to the important conductivity sum rule constraint. In view of the strong evidence for three dimensional (d) critical behavior [10–12] we do not restrict consideration to strictly 2d systems.

The inconsistencies which we aim to reconcile pertain to the behavior of the complex conductivity $\sigma = \sigma_1 + i\sigma_2$ and the diamagnetic susceptibility χ . [The widely discussed Nernst effect was examined in earlier work [13].] Below the transition, σ_2 directly relates to the superfluid density. If, above T_c , σ_2 were interpreted to reflect a remnant of the superfluid density (as expected in a simple fluctuation ([14] or more mesoscopic phase fluctuation theory [15, 16]), this would suggest a close relationship between σ_2 and the normal state diamagnetic susceptibility χ , which is not observed [3]. Problematic for a slightly different precursor scenario (the normal state vortex picture [17]) is the unexpectedly small (by two orders of magnitude [4]) value of the ratio of the real part of the conductivity σ_1 to χ in the normal state.

Our physical picture for the way in which transport is affected by preformed pairs is relatively simple to understand. In the presence of stronger than BCS attraction there are both fermionic and metastable Cooper pair degrees of freedom. The latter can be viewed as non-condensed pairs, or pair-correlated fermions. It can be seen from simple Boltzmann arguments [14] that bosons provide very large transport responses, provided they are in proximity to condensation. The Bose-Einstein distribution function which is then peaked at small wavevectors, is in stark contrast to its fermionic, Pauli principle restricted counterpart; it leads to a much stronger bosonic response to external field perturbations. Importantly, if one associates the pseudogap with long lived and meta-stable pairs in three dimensional systems, these enhancements, in transport can be shown to persist [13] to temperatures nearer to $T^* \gg T_c$, as one sees in a variety of different transport experiments. This should be distinguished from conventional fluctuation effects [16], which contribute in the critical regime very close to T_c .

In the usual BCS-like, purely fermionic Hamiltonian only fermions possess a hopping kinetic energy and and

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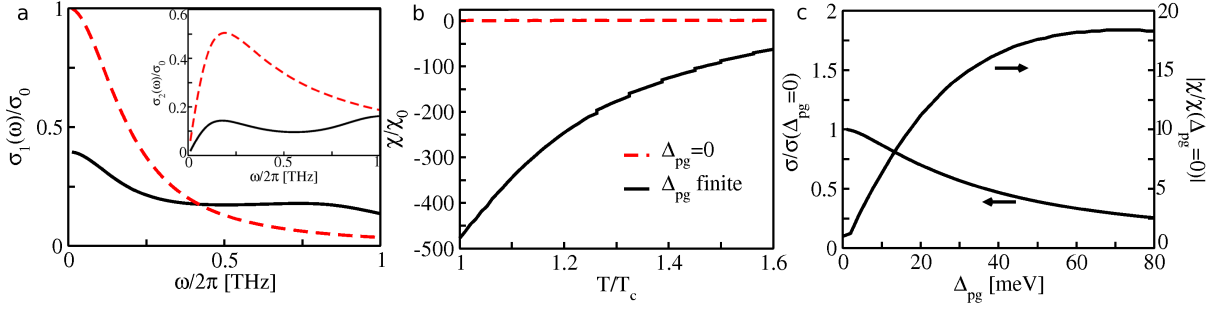


Figure 1: Schematic figures showing the effect for $T > T_c$ of the pseudogap on $\sigma(\omega)$ and χ^{dia} . (a) The real part of the conductivity σ_1 as a function of frequency with (black curve) and without (red dashed curve) the pseudogap. Inset: The imaginary part of the conductivity σ_2 as a function of frequency with (black curve) and without (red dashed curve) the pseudogap. (b) The diamagnetism with (black curve) and without (red dashed curve) the pseudogap. (c) The conductivity and diamagnetic susceptibility as functions of Δ_{pg} at $T = 15 \text{ meV}$.

thereby directly contribute to transport. The contribution to transport from pair correlated fermions enters indirectly by liberating these fermions through a break-up of the pairs. Technically, we can associate this coupling to fermionic transport as via the well known Aslamazov-Larkin diagram, importantly modified to include the self consistently determined fermionic pairing gap.

A stronger than BCS attractive interaction can be accommodated by a simple extension of Gor'kov theory. This leads to non-condensed pair effects [8] above and below T_c . Important here is the general form of the superconducting electromagnetic response which consists of three distinct contributions: (1) superfluid acceleration, (2) quasi-particle scattering and (3) pair breaking and pair forming. These all appear in conventional BCS superconductors, but at $T = 0$ this last effect is only present when there is disorder. However, in the presence of stronger than BCS attraction, and at $T \neq 0$, non-condensed pairs provide an alternate way to decrease the superfluid density, and the pair breaking and pair forming contributions will be concomitantly more prominent [18].

Without any detailed calculations we are now in a position to predict results associated with the THz conductivity and the diamagnetism, which will be supported by later microscopic theory. We now show how $\sigma_1(\omega \approx 0)$ is depressed by the presence of a pseudogap Δ_{pg} , how $\sigma_2(\omega)$ over a range of ω is also depressed while χ is greatly enhanced.

Fig.1(a) shows how the normal state $\sigma_1(\omega)$ (and in the inset $\sigma_2(\omega)$) behaves as a function of frequency. The red dashed curves are the results of conventional Drude theory. What happens when an above T_c pseudogap is present is shown by the black curves. The curves are normalized by σ_0 , the normal state value of the conductivity at $\omega = 0$ in Drude theory. Both theories (with or without the pseudogap) are consistent with the f-sum rule, and thus have the same fermionic carrier number $\left(\frac{n}{m}\right)_{xx}$. In the dc regime, with a pseudogap present, there are fewer fermions available to contribute to transport. Their number is reduced by the pseudogap. However, once the

frequency is sufficient to break the pairs into individual fermions, the conductivity rises above that of the Drude model. One can see that the effect of the pseudogap is to transfer the spectral weight from low frequencies to higher energies ($\omega \approx 2\Delta$, where Δ is the pairing gap, and $\Delta \equiv \Delta_{pg}$ above T_c). In this way one finds an extra “mid-infrared” contribution to the conductivity which is, as observed [19] strongly tied to the presence of a pseudogap.

The behavior of $\sigma_2(\omega)$, shown in the inset, is rather similarly constrained. On general principles, σ_2 must vanish at strictly zero frequency - as long as the system is normal. Thus both the red and black curves show that $\sigma_2(\omega \equiv 0) = 0$. Here one can see that the low frequency behavior is also suppressed by the presence of a pseudogap because of the gap-induced decrease in the number of carriers. Similarly, the second peak (around 2Δ) in $\sigma_1(\omega)$ leads, via a Kramers Kronig transform to a slight depression in $\sigma_2(\omega)$ in this frequency range. Hence as shown in the inset, $\sigma_2(\omega)$ is significantly reduced relative to the Drude result and tends overall to increase with ω . There is virtually no sign of a ω^{-1} upturn in σ_2 which would reflect a remnant of the superfluid density above T_c . This presumably is a fluctuation effect which pertains to the narrow critical regime.

In Fig.1(b) we present similar comparisons of the behavior of the orbital susceptibility above T_c in a non-gapped normal state (red dashed curve) and in the presence of a pseudogap (black curve). The curves are normalized by χ_0 , the absolute value of the diamagnetic susceptibility for $\Delta_{pg} = 0$ at $T = T_c$. One can see that in the absence of a pseudogap only a very weak Landau diamagnetism appears. However, the figure shows that in the presence of a pseudogap the diamagnetic contribution is significantly enhanced. This diamagnetism originates from the large electromagnetic response associated with bosonic degrees of freedom; the breaking of pairs allows this diamagnetism to be reflected in the fermionic response. It should be noted that Van Hove effects enhance this diamagnetism, as does d -wave pairing which

leads to an excess of low energy fermionic excitations. Moreover, this diamagnetism is not restricted to two dimensional models.

All of this leads to a simple anti-correlation between the dc conductivity and the diamagnetic susceptibility in the normal state, which is shown in Fig.1(c). Here we plot on the left and right hand axes the zero frequency conductivity as a function of varying pseudogap energy scale Δ_{pg} and the orbital (diamagnetic) susceptibility with varying Δ_{pg} respectively. The former is depressed as the pairing gap increases whereas the latter is enhanced.

These same conclusions (which are qualitatively compatible with experiment [3, 4, 17] derive from microscopic theory. Here the linear response of the electromagnetic current \mathbf{J} to a small vector potential \mathbf{A} is characterized by the tensor $\vec{\vec{P}} + \vec{\vec{n}}/m$ through the equation $\mathbf{J} = -(\vec{\vec{P}} + \vec{\vec{n}}/m)\mathbf{A}$. The transverse f-sum rule is an important constraint on any theory of transport

$$\lim_{\mathbf{q} \rightarrow 0} \int_{-\infty}^{\infty} \frac{d\omega}{\pi} \left(-\frac{\text{Im} P_{xx}(\mathbf{q}, \omega)}{\omega} \right) = \left(\frac{n_n}{m} \right)_{xx} \quad (1)$$

where $\vec{\vec{n}}_n/m$ is the normal fluid density and P_{xx} is the diagonal component of the paramagnetic current, $\vec{\vec{P}}$, along the x-direction. Similarly, $(n/m)_{xx}$ is the diagonal component of $\vec{\vec{n}}/m$ along the x-direction. We stress that only the fermionic density (and mass) appears on the right hand side of Eq. (1). The sum rule establishes a strong connection between transport and the fermionic kinetic energy, so that many body interactions only serve to redistribute the spectral weight. Thus, for example, even though meta-stable pairs are present, their contribution to transport is indirect and appears when such pairs can be decomposed. This version of the f-sum rule applies to any many body Hamiltonian which contains an arbitrary two body interaction and a kinetic energy associated with fermions.

Throughout, we work in the transverse gauge. As a consequence all effects of the order parameter collective modes (which are longitudinal) do not enter. The complex conductivity is microscopically defined in terms of $\vec{\vec{P}}$ and $\vec{\vec{n}}/m$:

$$\sigma(\omega) = -\lim_{\mathbf{q} \rightarrow 0} \frac{P_{xx}(\mathbf{q}, \omega) + (n/m)_{xx}}{i\omega} \quad (2)$$

Above T_c , the linear diamagnetic response is similarly related to P_{xx} and $(n/m)_{xx}$. It is given by

$$\chi_{zz}^{\text{dia}} = -\lim_{q_y \rightarrow 0} \left[\frac{\text{Re} P_{xx}(\mathbf{q}, \omega = 0) + (n/m)_{xx}}{q_y^2} \right]_{q_x=q_z=0} \quad (3)$$

In the superfluid phase the tensors $\vec{\vec{P}}$ and $\vec{\vec{n}}/m$ no longer cancel when $\mathbf{q} \rightarrow 0$, reflecting the Meissner effect. We stress that Equations (1)-(3) are completely general.

We now turn to more microscopic calculations. Previous papers [8, 20] have described how the parameters $\Delta(T)$, $\Delta_{pg}(T)$, and μ are self consistently obtained and how one accommodates a variety of dopings, by effectively fitting the attractive interaction to match T^* and T_c . Our figures correspond to moderate underdoping. A nearest neighbor tightbinding dispersion $\xi_{\mathbf{p}} = -2t[\cos(k_x a) + \cos(k_y a)] - \mu$ with $t = 300 \text{ meV}$ is used throughout, and for simplicity, we took a simple $\tau \propto T^{-2}$ power law (associated with the Fermi arcs [21]) for the transport lifetime. Very few of our results depended on this assumption which was made in earlier work [18]. A general finding is that, while Δ_{pg} decreases monotonically from T_c to T^* , with decreasing T below T_c , Δ_{pg} decreases while Δ_{sc} rises, reflecting the fact that finite momentum pairs are converted to the $q = 0$ condensate, while maintaining an overall nearly constant $\Delta(T)$. We have previously derived microscopic representations of $\vec{\vec{P}}$ and $\vec{\vec{n}}/m$ [18, 20, 22]. Importantly, our gauge invariant electromagnetic response function analytically satisfies the transverse f-sum rule. One can derive these contributions in a variety of ways but the most straightforward involves inclusion of generalized Maki-Thompson and Aslamazov-Larkin diagrams. The latter are considered to be effectively equivalent to Boltzmann (or time dependent Ginsburg-Landau-like) approaches to bosonic transport. Here the stronger-than-BCS attraction enters in an important way in order to insure that the pairing gap energy scale Δ is explicitly incorporated. The paramagnetic tensor current-current correlation function $\vec{\vec{P}}$ is

$$P_{xx}(\mathbf{q}, \omega) = \sum_{\mathbf{p}} \frac{\partial \xi_{\mathbf{p}}}{\partial p_x} \frac{\partial \xi_{\mathbf{p}}}{\partial p_x} \left[\frac{E^+ + E^-}{E^+ E^-} \frac{E^+ E^- - \xi^+ \xi^- - \Delta_{sc}^2 + \Delta_{pg}^2}{\omega^2 - (E^+ + E^-)^2} (1 - f(E^+) - f(E^-)) \right. \\ \left. - \frac{E^+ - E^-}{E^+ E^-} \frac{E^+ E^- + \xi^+ \xi^- + \Delta_{sc}^2 - \Delta_{pg}^2}{\omega^2 - (E^+ - E^-)^2} (f(E^+) - f(E^-)) \right] \quad (4)$$

where ω has a small imaginary part and f the Fermi func-

tion. Here $E_{\mathbf{p}} \equiv \sqrt{\xi_{\mathbf{p}}^2 + \Delta^2(T)}$, where $\xi_{\mathbf{p}} = \epsilon_{\mathbf{p}} - \mu$, and

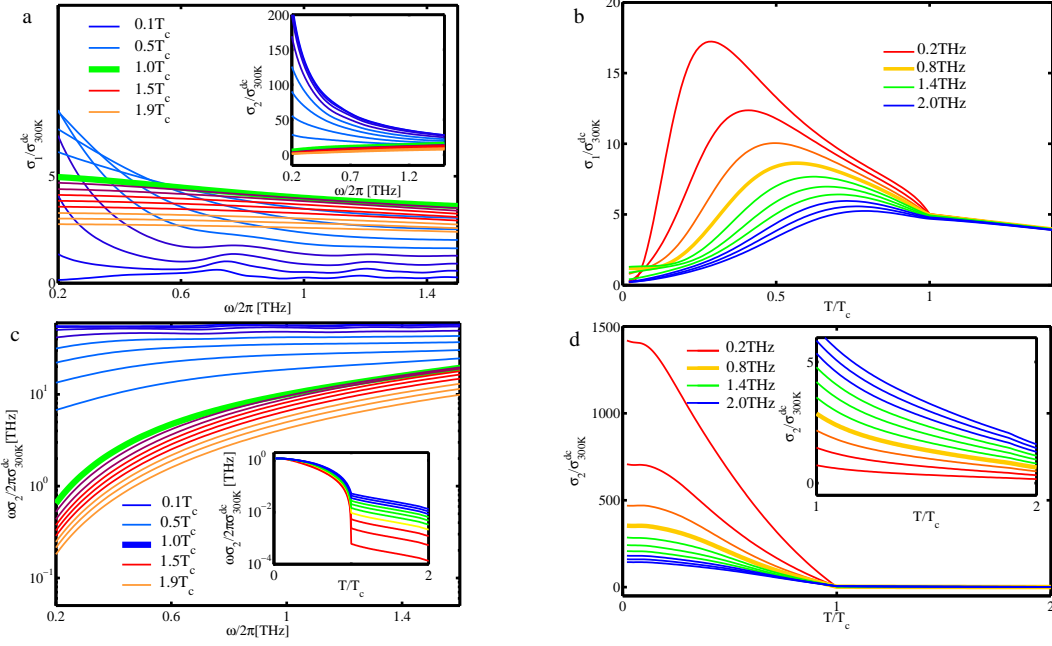


Figure 2: Results for σ_1 and σ_2 . (a) σ_1 as a function of frequency. Inset: σ_2 as a function of frequency. (b) σ_1 as a function of temperature. (c) $\omega\sigma_2$ as a function of frequency. Inset: $\omega\sigma_2$ as a function of temperature. (d) σ_2 as a function of temperature. Inset: σ_2 as a function of temperature near T_c .

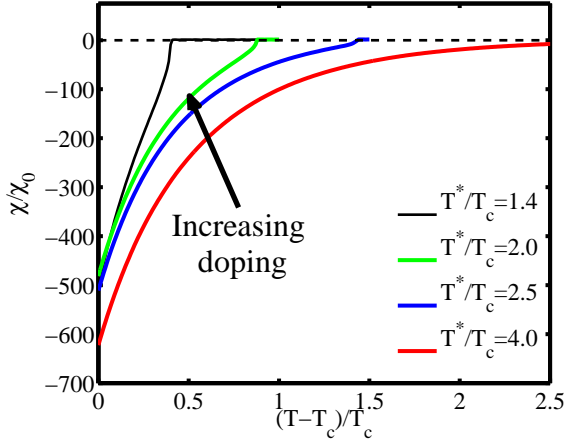


Figure 3: The diamagnetism as a function of temperature for different “hole concentrations” as parameterized via the indicated T^*/T_c .

Δ_{sc} (Δ_{pg}) is the gap component of the condensed (non-condensed) pairs, with $\Delta = \sqrt{\Delta_{sc}^2 + \Delta_{pg}^2}$. All transport expressions in this paper reduce to those of strict BCS theory when the attraction is weak and $\Delta_{pg} = 0$. Here we define $E^\pm = E_{\mathbf{p} \pm \mathbf{q}/2}$ and $\xi^\pm = \xi_{\mathbf{p} \pm \mathbf{q}/2}$. Importantly, the terms on the first line in Eq.(4) represent the pair breaking and pair forming contributions. The second line is associated with fermionic scattering. Also important to the electromagnetic response is the number density \tilde{n}/m

which can be rewritten as

$$= 2 \sum_{\mathbf{p}} \frac{\partial \xi_{\mathbf{p}}}{\partial p_x} \frac{\partial \xi_{\mathbf{p}}}{\partial p_x} \left[\frac{\Delta_{sc}^2 + \Delta_{pg}^2}{E_p^2} \left(\frac{1 - 2f(E_p)}{2E_p} \right) - f'(E_p) \right]. \quad (5)$$

Eqs.(2)-(3) yield analytic expressions for $\sigma(\omega)$ and χ^{dia} . Fig.2 displays our more quantitative results for σ_1 and σ_2 as both functions of ω and T . The layout is designed to duplicate figures from Ref. 3 and the general trends are similar. Thus one sees from Fig.2(a) and its inset that well above T_c , the real part of the conductivity is almost frequency independent. The imaginary part is small in this regime. At the lowest temperatures σ_1 contains much reduced spectral weight while the frequency dependence of $\sigma_2 \propto \omega^{-1}$; both of these reflect the characteristic behavior of a superfluid.

Here as in the experimental studies [3], we focus primarily on the temperature dependent plots in Figs.2(b), (d) and the inset to (c). One sees that σ_1 shows a slow decrease as the temperature is raised above T_c . Somewhat below T_c , σ_1 exhibits a peak which occurs at progressively lower temperatures as the probe frequency is decreased. Roughly at T_c we find that σ_2 shows a sharp upturn at low ω . The region of finite σ_2 above the transition can be seen from the inset in Fig.2(d) and it is clearly very small in the pseudogap state. The inset of Fig.2(d) shows an expanded view of $\sigma_2(T)$ near T_c . In agreement with experiment, the nesting of the σ_2 versus T curves switches orders above T_c . This important point reflects the fact that $\sigma_2(\omega)$ is generally increasing with increasing ω above T_c as seen in the inset in Fig. 1(a) and in exper-

iment. This is in contrast to the behavior expected of a fluctuation contribution where a ω^{-1} dependence would occur. However, in slightly different plots, the counterpart experimental studies reveal a small 10-15K range where this fluctuation contribution is visible. This effect would not be present in a mean field approach. As speculated in Ref. 3, one should distinguish these near T_c critical fluctuations from preformed pairs which persist to much higher temperatures.

These effects are made clearer by plotting the “phase stiffness” which is proportional to the quantity $\omega\sigma_2$ and is shown in Fig.2(c). Deep in the superconducting state there is no ω dependence to $\omega\sigma_2(\omega)$, while at higher T this dependence becomes apparent. In the inset to (c), the temperature dependence of $\omega\sigma_2(T)$ is displayed. We see that above T_c , $\omega\sigma_2$ is never strictly constant, as would be expected from fluctuation contributions. In experiment the onset of finite frequency spreading of the curves at $T \leq T_c$ has been attributed to Kosterlitz-Thouless physics [23].

Finally, we turn to the diamagnetic response. Figure 3 shows χ^{dia} as a function of temperature for four different dopings. Independently of the particular parameters that are used, it is seen that the magnitude of χ^{dia} is enhanced even at temperatures well above T_c . We should not associate this diamagnetism with short range Meissner currents, as might be appropriate to alternative phase fluctuation [15] or normal state vortex scenarios [4]. Rather here, the diamagnetism arises from the large contribution of non-condensed pairs which are in proximity to condensation [13, 14]. This has a similarity to low d fluctuation effects, but arises in the 3d systems here from stronger than BCS attraction, which

stabilizes these pair degrees of freedom. Since the kinetic energy ultimately resides in the fermionic system, it is not surprising that we find it is the pair breaking terms which provide the conduit for communicating enhanced bosonic transport contributions to the fermionic transport channel. Because we are working at effectively zero magnetic field, we have not addressed diamagnetism associated with non-linear response, although this appears to be very anomalous experimentally [17]. At the leading order level we have shown here that there is a profound connection between the complex conductivity and this orbital magnetism. Importantly, while the preformed pairs in our theory give a large contribution to the diamagnetic susceptibility, as is observed experimentally, the imaginary part of the conductivity is suppressed to zero much closer to T_c , as observed.

We end with the following observations. Our theoretical approach, has virtually no flexibility; it was set up [20, 22] before there was much experimental interest in these transport measurements. In accord with experiment, we find: (i) that pseudogap effects lead to an enhanced diamagnetism above T_c , (ii) that the imaginary conductivity $\sigma_2(\omega)$ is reduced to zero in a very narrow range of T above T_c , and (iii) that the real conductivity $\sigma_1(\omega \approx 0)$ is suppressed as the pseudogap becomes larger. This last point is in turn associated with a transfer of low ω conductivity spectral weight into the mid-infrared region.

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